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A method to predict two-phase pressure drop using condensation heat transfer data

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Abstract — The data of Kutateladze related to mean condensation heat transfer coefficients of pure vapors of steam flowing in a horizontal tube are correlated considering the phenomenon as a homogenous flow. Colburn's analogy assuming to hold good the mean friction coefficients for a homogenous flow conditions are evaluated making use of heat transfer data. By extending the model further the two-phase frictional pressure drop multipliers are established. Predictions from these correlations have reasonably agreed with the magnitudes obtained from Lockhart and Martinelli correlations which are conventionally used in the design. © 2000 Éditions scientifiques et médicales Elsevier SAS

two-phase flow / condensation heat transfer analogy / friction multipliers

Nomenclature

A	cross section area of the tube	m^2
A_0	constants in equation (6)	
C	constant in equations (1) and (3)	
C_p	specific heat at constant pressure	$\mathrm{J\cdot kg^{-1}\cdot K^{-1}}$
D	diameter of the tube	m
f_{TPF}	two-phase friction coefficient	
g	acceleration due to gravity	$\mathrm{m\cdot s^{-2}}$
h	mean heat transfer coefficient	$\mathrm{W\cdot m^{-2}\cdot K^{-1}}$
h_{fg}	latent heat of vaporisation	$\mathrm{J\cdot kg^{-1}}$
k_{L}	liquid thermal conductivity	$\mathrm{W\cdot m^{-1}\cdot K^{-1}}$
L	length of the tube	m
$\dot{m}$	mass flow rate per unit area	$\mathrm{kg\cdot s^{-1}}$
n	index in equation (1)	
Nu_m	average Nusselt number, $h_m D/k_{\mathrm{L}}$	
Pr	Prandtl number, $\mu C_p/k_{\mathrm{L}}$	
P	pressure absolute	$\mathrm{N\cdot m^{-2}}$
q	heat flux	$\mathrm{W\cdot m^{-2}}$
Re	Reynolds number, $4\dot{m}/(\pi D\mu)$ or $4qL/(\mu_{\mathrm{L}}h_{\mathrm{fg}})$	
St	Stanton number, $Nu/(Re_{\mathrm{L}}Pr)$	
T	temperature	K
U	velocity of flow	$\mathrm{m\cdot s^{-1}}$
X_{tt}	Martinelli parameter	
x	dryness fraction by weight	
z	distance measured along the condenser tube	m

Greek symbols

μ	dynamic viscosity of condensate	$\mathrm{N\cdot m^{-1}\cdot s^{-1}}$
ν	kinematic viscosity	$\mathrm{m^2\cdot s^{-1}}$
ρ	density	$\mathrm{kg\cdot m^{-3}}$
τ	shear stress	$\mathrm{N\cdot m^{-2}}$
ϕ	two-phase friction multiplier in equations (12) and (15)	

Subscripts

L	liquid
f	friction
v	vapor
TP	two-phase
TPF	two-phase friction

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1. INTRODUCTION

Condensation of flowing vapor inside a tube has become a topic of investigation with the advent of modern refrigerating systems. Unlike the case of condensation of pure stagnant vapor on a surface as solved by Nusselt [1], the phenomenon of condensation of vapors flowing in a tube is more complex because of several hydrodynamic flow patterns that may arise all along the length of the tube.

Some of the important investigations, both theoretical and experimental investigations, are due to Kutateladze [2], Chitti and Anand [3], Lu and Suryanarayana [4], Rifert [5, 6], Soliman et al. [7], Tepe and Mueller [8], Rufer and Kezios [9], Akers et al. [10], Ananiev et al. [11], Shah [12] and Kunz et al. [13].

The survey reveals that some of the correlations are obtained purely by dimensional analysis, while other investigators worked out the problem employing a theoretical and semi-theoretical approach using two-phase flow characteristics. By and large the annular two-phase flow regime is considered in modelling with the interfacial shear stress and void fraction etc., being estimated from air–water mixtures flow studies of Lockhart and Martinelli [14], which are mainly for adiabatic flow conditions.

Owens [15] employed homogenous flow model in the estimation of two-phase pressure drop over a channel with boiling taking place in it. He used the friction factor of single-phase liquid flow for the estimation of two-phase pressure drop. The pressure drop predictions from his analysis are in good agreement with the data of Schrock and Grossman [16]. A similar analysis of Mendlers [17] using single-phase friction factor predicts well the pressure drop predictions of two-phase flows at 800, 1 200 and 1 600 psia. In two-phase flows of condensing vapors the momentum change associated with phase transformation could be significant. In the estimation of the two-phase pressure drop the friction coefficient can be very much different from liquid phase friction component. Hence, its accurate estimation depends upon the correctness of the two-phase friction coefficient value chosen in computations. In studies related to single-phase flow with convective heat transfer in circular tubes, the principle of analogy is well accepted. In the present study of condensation of pure vapors in a horizontal tube the data would be treated considering it as homogenous flow phenomenon with physical properties such as density and viscosity of the mixture defined in an appropriate manner as functions of dryness fraction by weight. In the correlation developed by Soliman et al. [7], the estimation of local heat transfer coefficients is based on two phase separated flow model. Basically, the estimation of interfacial shear stress between the two phases is made using Lockhart and Martinelli correlations developed for the flow of air–water mixture under adiabatic conditions.

The purpose of this study is to test the applicability of the homogenous model concept. The condensation heat transfer results are also used in the evaluation of two-phase pressure drop multipliers which are compared with the predictions of Lockhart and Martinelli [12]. The agreement is found to be satisfactory. Thus, the aim of the article is to use the heat transfer data in the estimation of momentum transfer characteristics.

2. MEAN CONDENSATION HEAT TRANSFER COEFFICIENT

Kutateladze [2] reported experimental data for the condensation of steam in a horizontal tube. His data cover wide ranges of L/D ($117 < L/D < 400$), system pressures ($22 < P < 97$ bar) and condensate Reynolds number at the exit of the condenser ($3\,000 < Re_{\mathrm{L}} < 1.5{\cdot}10^5$). In the first stage of analysis a dimensionless correlation is attempted between j_{TP} factor and Re_{L} defined as in equations (1) and (2). It is observed that for each L/D, the data could be successfully correlated in the form

$$j_{\mathrm{TP}} = C/Re_{\mathrm{L}}^{n} \qquad (1)$$

where the j_{TP} factor is tentatively defined as

$$j_{\mathrm{TP}} = \frac{Nu}{Re_{\mathrm{L}} Pr} Pr^{2/3} \qquad (2)$$

The dimensionless terms appearing in equations (1) and (2) are defined as follows:

$$Re_{\mathrm{L}} = \frac{4\dot{m}}{\pi D \mu_{\mathrm{L}}} \quad \text{or} \quad \frac{4q_{\mathrm{L}}}{\mu_{\mathrm{L}} f_{\mathrm{fg}}}$$

$$Pr = \frac{\mu_{\mathrm{L}} C_p}{k_{\mathrm{L}}}, \qquad Nu = \frac{hD}{k_{\mathrm{L}}}$$

Variation of j_{TP} as function of Re_{L} is shown in *figure 1*.

Evidently, the data could be satisfactorily represented in terms of the dimensionless terms chosen.

However, the coefficient and the exponent C and n, respectively, in equation (1) are found to be varying as per the entries shown in *table I* from the regression analysis. These values C and n shown in *table I* are further con-

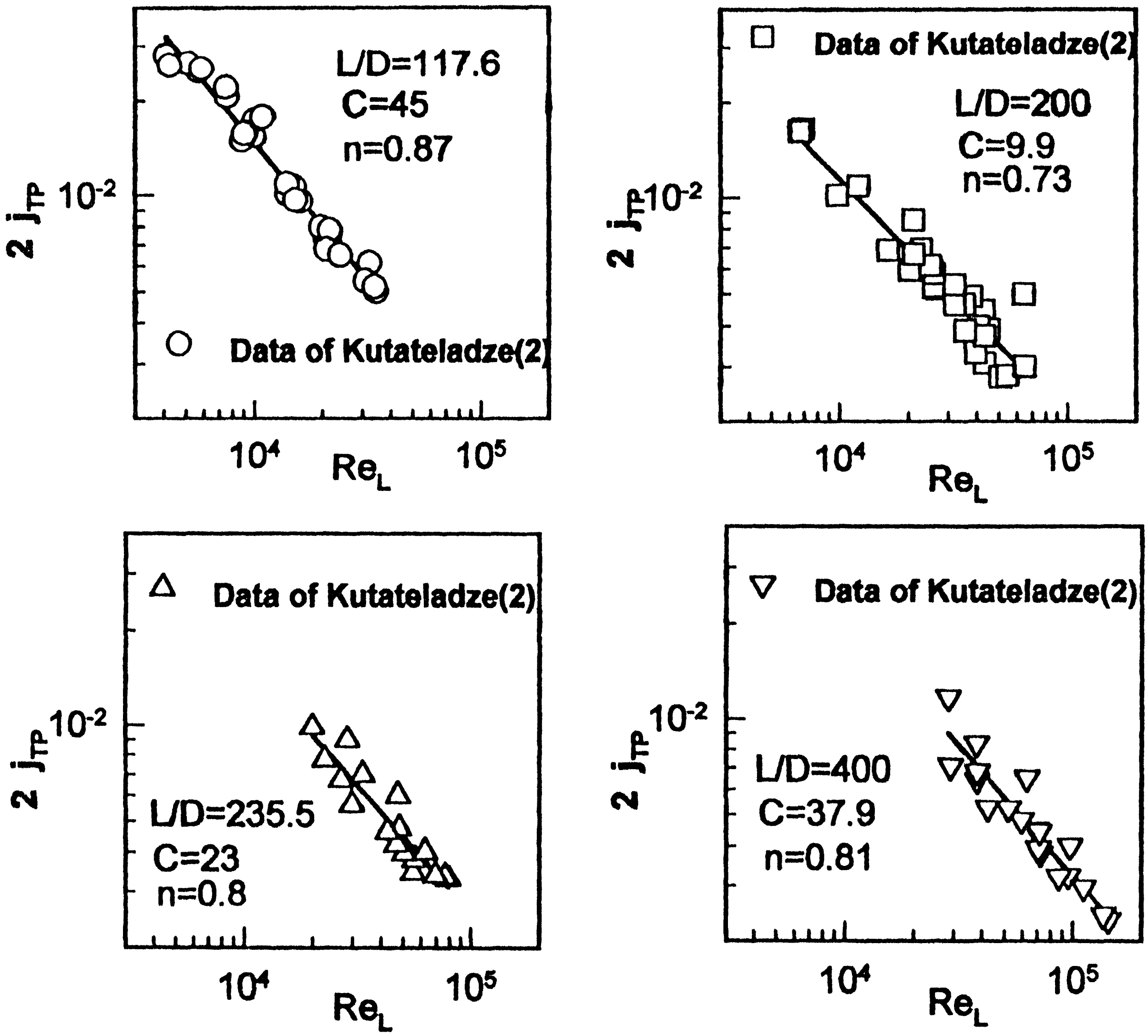

Figure 1. Condensation data of Kutateladze (2)—j_{TP} versus Reynolds number of condensate at exit.

sidered to be functions of L/D and can be fairly represented by a third-degree polynomial as follows:

$$C = 840 - 11.01\left(\frac{L}{D}\right) + 0.0463\left(\frac{L}{D}\right)^2 - 0.59\cdot10^{-4}\left(\frac{L}{D}\right)^3 \qquad (3)$$

$$n = 2.42 - 0.023\left(\frac{L}{D}\right) + 0.97\cdot10^{-4}\left(\frac{L}{D}\right)^2 - 1.25\cdot10^{-7}\left(\frac{L}{D}\right)^3 \qquad (4)$$

TABLE I
Values of C and n deduced from the curves of *figure 1* for various values of L/D.

S. No.	L/D	C	n
1	117.6	45.0	0.87
2	200.0	9.90	0.73
3	235.0	23.0	0.80
4	400.0	37.9	0.81

3. TWO-PHASE FLOW MODEL AND PRESSURE DROP

It is assumed further that there can be a relationship between j_{TP} factor and friction coefficient f_{TPF} for two phase flows similar to the one available for single phase convective heat transfer, i.e., similar to Colburn analogy.

Thus, as per the tentative assumption of the homogeneous flow model hereafter

$$j_{\mathrm{TP}} = \frac{f_{\mathrm{TPF}}}{2} \tag{5}$$

where f_{TPF} is the two-phase friction coefficient.

The validity of the relationship will be checked subsequently in the foregoing analysis. In order to accomplish the objective it is further assumed that the local two-phase friction coefficient is the average of the local friction coefficient of the lighter and heavier phases at any given location of the condenser tube, i.e., for a given dryness fraction x by weight.

Thus,

$$f_{\mathrm{TPF}}(x) = \frac{A_0}{2}\left\{\frac{(1-x)^{1-m}}{Re_{\mathrm{L}}^m} + \frac{x^m}{Re_{\mathrm{v}}^m}\right\} \tag{6}$$

where A_0 and m are constants and

$$Re_{\mathrm{L}} = \frac{4\dot{m}}{\pi D \mu_{\mathrm{L}}}, \qquad Re_{\mathrm{v}} = \frac{4\dot{m}}{\pi D \mu_{\mathrm{v}}}$$

Equation (6) can be simplified as follows:

$$f_{\mathrm{TPF}}(x) = \frac{A_0}{2Re_{\mathrm{L}}^m}\left\{(1-x)^{1-m} + x^{1-m}\phi_\mu^m\right\} \tag{7}$$

where

$$\phi_\mu = \frac{\mu_{\mathrm{v}}}{\mu_{\mathrm{L}}}$$

In equation (7), A_0 and m are yet to be determined and to evaluate these values the following condition is employed:

$$f_{\mathrm{TPF}} = \int_0^1 f_{\mathrm{TPF}}(x)\,\mathrm{d}x \tag{8}$$

Substitution of equations (5) and (6) in equation (8) will yield the following conditions after performing integration:

$$A_0 = \frac{2(2-n)C}{1+\phi_\mu^n} \tag{9}$$

and $m = n$.

The estimation of two-phase frictional pressure drop component can be undertaken with the available information for any dryness fraction x as follows.

The Fanning friction coefficient relationship for homogenous model can be written as follows:

$$\frac{\Delta P_{\mathrm{TPF}}}{\Delta L} = \frac{4 f_{\mathrm{TPF}}(x)}{2D}\left[\frac{\dot{m}}{\rho_{\mathrm{TP}} A}\right]^2 \rho_{\mathrm{TP}} \tag{10}$$

where $A = \pi D^2/4$ (total flow area of tube) and

$$\frac{1}{\rho_{\mathrm{TP}}} = \frac{1-x}{\rho_{\mathrm{L}}} + \frac{x}{\rho_{\mathrm{v}}} \tag{11}$$

The analysis is further validated by comparing equation (10) of the homogenous model with the separated flow model of Lockhart and Martinelli [14]. The two-phase frictional pressure drop according to them can be estimated by the following relationships for the turbulent–turbulent regimes of the two phases:

$$\frac{\Delta P_{\mathrm{TPF}}}{\Delta P_{\mathrm{v}}} = \phi_{\mathrm{v}}^2 \tag{12}$$

and

$$\phi_{\mathrm{v}} = 1 + 2.85 X_{\mathrm{tt}}^{0.523} \tag{13}$$

where

$$X_{\mathrm{tt}} = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{v}}}\right)^{0.1}\left(\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{L}}}\right)^{0.5} \tag{14}$$

Further,

$$\frac{\Delta P_{\mathrm{TPF}}}{\Delta P_{\mathrm{L}}} = \phi_{\mathrm{L}}^2 \tag{15}$$

and

$$\phi_{\mathrm{L}} = \frac{\phi_{\mathrm{v}}}{X_{\mathrm{tt}}^{0.90}} \tag{16}$$

The friction multiplier ϕ_{v} of the present analysis can be computed with the aid of the following equation:

$$\phi_{\mathrm{v}}^2 = \frac{\Delta P_{\mathrm{TPF}}}{\Delta P_{\mathrm{v}}} = \frac{\Delta P_{\mathrm{TPF}}}{\Delta P_{\mathrm{L}}}\left(\frac{\Delta P_{\mathrm{L}}}{\Delta P_{\mathrm{v}}}\right) = \phi_{\mathrm{L}}^2 X_{\mathrm{tt}}^2$$

In *figures 2–4* the results of the present analysis are compared with the predictions from the empirical correlations of Lockhart and Martinelli [14]. Evidently, for different system conditions, the agreement between the two analyses is found to be satisfactory. Nevertheless, the coefficient C in equation (3) is adjusted marginally as function of the pressure so that it agrees better with the

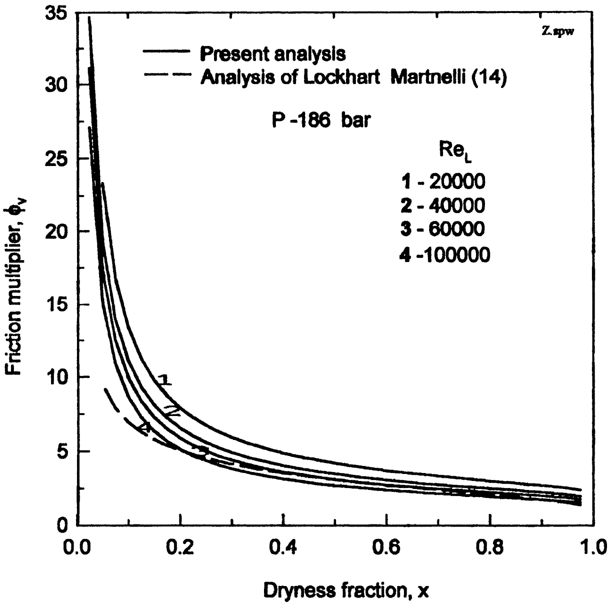

Figure 2. Effect of Reynolds number on friction multiplier ϕ_v.

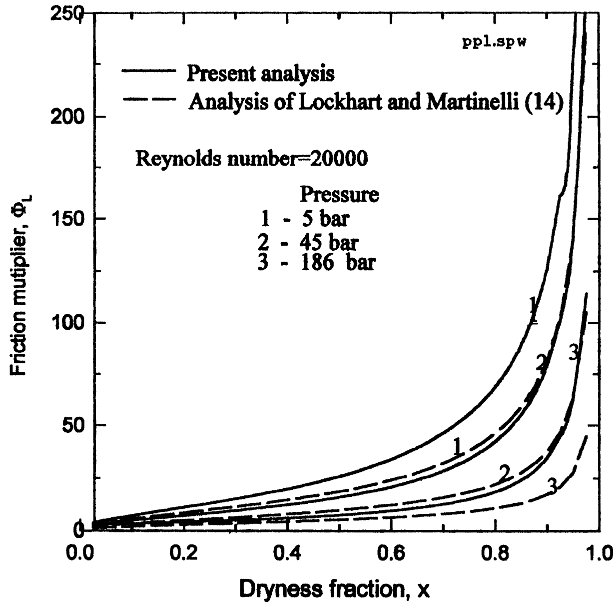

Figure 3. Variation of friction multiplier ϕ_L with dryness fraction.

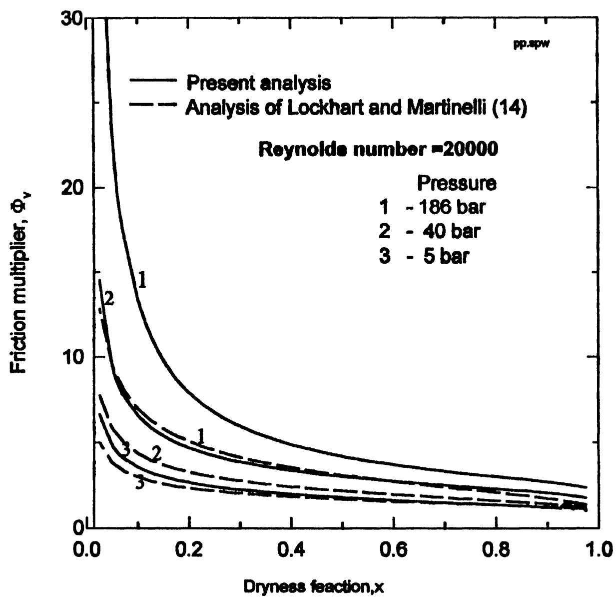

Figure 4. Variation of friction multiplier ϕ_v with dryness fraction.

Lockhart and Martinelli correlations. These new values of C do not significantly alter the plots in *figure 1* except for a slight drift in the regression lines passing through the data points. Thus, the modified form of the constant C hereafter can be taken by the relationship

$$C = B(P) - 11.0\left(\frac{L}{D}\right) + 0.0463\left(\frac{L}{D}\right)^2 - 0.59\cdot 10^{-4}\left(\frac{L}{D}\right)^3 \tag{17}$$

where

$$B(P) = 825 + 0.755P - 0.0023P^2 \tag{18}$$

Thus, for steam-water system value of $B(P)$ varies from 826 to 880 respectively at $P = 1$ bar to $P = P_c$ critical. It can be observed that the dependence of average and local friction coefficient with L/D is consistent with the observations made by Linehan et al. [19]. Further, verification of the hypothesis is accomplished making use of the air–water data of Lockhart and Martinelli [14] and Jenkins [18] with estimates of ϕ_v made from the present analysis. It can be seen that from *figure 5* the present theory favourably agrees with the spectrum of the data over ranges of Re_L investigated by them.

However, the present analysis differs from that of Lockhart and Martinelli [14] in as much as ϕ_L and ϕ_v are also found to be dependent on the Reynods number of the condensate at the exit. Thus, the present approach makes use of condensation heat transfer data in the estimation of two-phase momentum transport characteristics.

4. CONCLUSIONS

The present article provides a correlation for predicting two-phase friction coefficient from the condensation

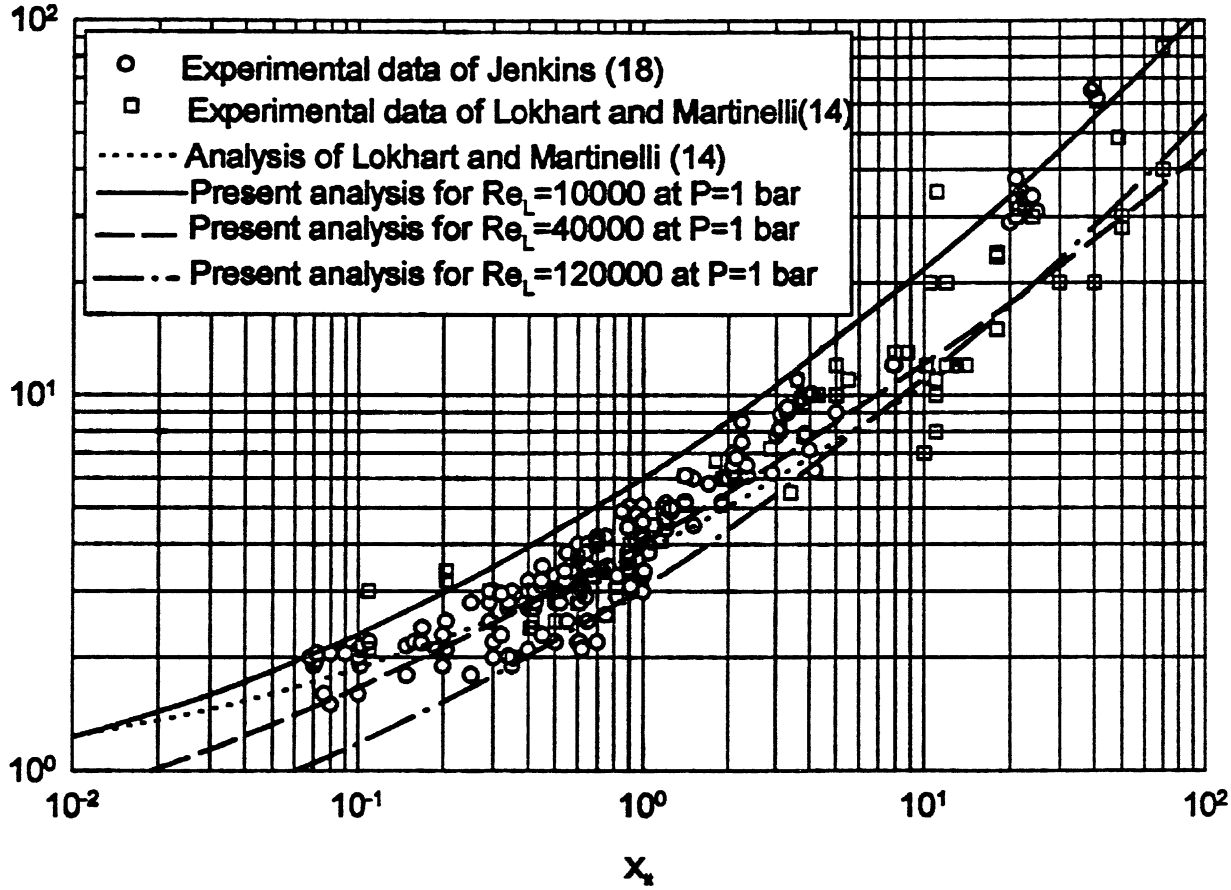

Figure 5. Relation between ϕ_v and parameter X_{tt}—comparison with experimental data.

heat transfer data for a wide range of parameters. It is observed that the present heat transfer correlation agrees satisfactorily with the data of Kutateladze within error of $\pm 20\,\%$. Further, the multipliers as proposed herein can be used for the prediction of frictional pressure drop at least for the case of steam condensing at high pressure. In other words, the condensation heat transfer data could be indirectly utilised in the estimation of two-phase friction factor which is needed for the evaluation of two-phase pressure drop under diabatic conditions.

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REFERENCES

[1] Nusselt W., The surface condensation of water vapor, Z. Verb. Dt.sd Ch. Ing. 6 (1916) 541–546.

[2] Kutateladze S.S., Aspects of Heat Transfer and Hydraulics of Two-Phase Medium (in Russian), Government Energy Publishing House, Moscow, 1961.

[3] Chitti M.S., Anand N.K., An analytical model for local heat transfer coefficients for forced convective condensation inside smooth horizontal tubes, Int. J. Heat Mass Tran. 38 (1995) 615–627.

[4] Lu Q., Suryanarayana N., Condensation of a vapor flowing inside a horizontal rectangular duct, J. Heat Tran., T. ASME 117 (1995) 418–424.

[5] Rifert V.G., Heat transfer and flow modes of phases in laminar film vapour condensation inside a horizontal tube, Int. J. Heat Mass Tran. 31 (1983) 517–523.

[6] Rifert V.G., Vapor condensation inside horizontal tubes, J. Engrg. Phys. 44 (1983) 1017–1029.

[7] Soliman M., Schuster J.R., Berenson P.J., A general heat transfer correlation for annular flow condensation, J. Heat Tran., T. ASME 90 (1968) 267–276.

[8] Tepe J.B., Mueller A.C., Condensation and subcooling inside an inclined tube, Chemical Engineering Progress 43 (1947) 267–278.

[9] Rufer C., Kezios S.P., Analysis of two-phase, one component stratified flow with condensation, J. Heat Tran. 88 (1966) 265–275.

[10] Akers W.W., Rosson H.E., in: ASME, AICHE, 3rd National Heat Transfer Conference, Storrs, CT, USA, 1959.

[11] Ananiev E.P., Boyko L.D., Kruzhilin G.N., Heat transfer in the presence of steam condensation in a horizontal tube, in: Int. Developments in Heat Transfer, Part 2, 1961, pp. 290–295.

[12] Shah M.M., A general correlation for heat transfer during film conden-sation inside pipes, Int. J. Heat Mass Tran. 22 (1979) 547–556.

[13] Kunz H.R., Yerazunis S., An analysis of film condensation, film evaporation and single phase heat transfer, in: Heat Transfer Conference, Seattle, WA, Paper 67-H.T.-1, 1967.

[14] Lockhart W., Martinelli R.C., Proposed correlation of data for isothermal two-phase, two-component flow in pipes, Chemical Engineering Progress 45 (1947) 39–46.

[15] Owens W.L., Two-phase pressure gradient, in: Int. Developments in Heat Transfer, Part 2, 1961, pp. 363–368.

[16] Schrock V.E., Grossman L.M, Forced convection boiling studies, C.I.E.R. Report Series, 2, 1959.

[17] Mendeler O.J., Natural-circulation tests with water at 800 to 200 psia under non boiling, local boiling and bulk boiling conditions, J. Heat Tran. 83 (1961) 261–273.

[18] Jenkins R.M.Ch.E., M.S. Thesis, University of Del, 1947.

[19] Linehan J.H., Patrick M., El-Wakil M.M., On the interface shear stress on annular flow condensation, J. Heat Tran., T. ASME 91 (1969) 450–452.